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# ON THE ADDITIVITY OF TENSORS OF STRAINS AND DISPLACEMENTS FOR FINITE ELASTOPLASTIC DEFORMATIONS

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A relation connecting the components of the plastic, elastic and total deformation tensors is given in [1]. The author of [2] asserts that "the usual assumption that the total deformation is a sum of the elastic and plastic components, ceases to be valid when the deformation is finite". This assertion is based on the fact that "the components of the finite deformation are expressed in terms of the displacements in a non-linear manner and cannot, generally speaking, be additive". Below it is shown that for a simple process the property of additivity of the deformation tensors (which is true according to [1] for the covariant components of the plastic, elastic and total deformation tensors) implies, for a finite homogeneous deformation, the additivity of the displacements corresponding to the plastic, elastic and total displacements.

Following [1], we shall consider a deformable continuum and single out its three states: the initial, the deformed and the "unloading" state. The basis vectors $\vartheta_i$ of the Lagrangian $\xi_i$ -coordinate system, the components of the metric tensors $g_{ij}$ and other characteristics related to the above three states will be denoted by the superscripts $^\circ$, $\hat{}$ and $*$, respectively. We carry out the analysis with the help of a two-stage deformation: from the state $g_{ij}^\circ$ to the state $g_{ij}^*$, and from this into the state $g_{ij}^\wedge$. The corresponding passages are determined by the plastic $\varepsilon_i^{pj}$, elastic $\varepsilon_i^{ej}$ and the total $\varepsilon_i^j$ deformation

$$2\varepsilon_{ij}^p = g_{ij}^* - g_{ij}^\circ, \quad 2\varepsilon_{ij}^e = g_{ij}^\wedge - g_{ij}^*, \quad 2\varepsilon_{ij} = g_{ij}^\wedge - g_{ij}^\circ \tag{1}$$
$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p$$

The metric spaces of the three states are connected with each other by the following relations:

$$\vartheta_i^\wedge = (\delta_i^k + \nabla_i^\circ u^k)\,\vartheta_k^\circ = c_i^{\circ k}\vartheta_k^\circ, \quad \vartheta_i^* = (\delta_i^k + \nabla_i^\circ u^{pk})\,\vartheta_k^\circ = c_i^{\circ pk}\vartheta_k^\circ \tag{2}$$
$$\vartheta_i^\wedge = (\delta_i^k + \nabla_i^* u^{ek})\,\vartheta_k^* = c_i^{*ek}\vartheta_k^*, \quad \vartheta_i^\circ = (\delta_i^k + \nabla_i^\wedge u^k)\,\vartheta_k^\wedge = c_i^{\wedge k}\vartheta_k^\wedge$$
$$\vartheta_i^\circ = (\delta_i^k - \nabla_i^* u^{pk})\,\vartheta_k^* = c_i^{*pk}\vartheta_k^*, \quad \vartheta_i^* = (\delta_i^k - \nabla_i^\wedge u^{ek})\,\vartheta_k^\wedge = c_i^{\wedge ek}\vartheta_k^\wedge$$
$$g_{ij}^\wedge = c_i^{\circ k} c_j^{\circ n} g_{kn}^\circ, \quad g_{ij}^* = c_i^{\circ pk} c_j^{\circ pn} g_{kn}^\circ, \quad g_{ij}^\wedge = c_i^{*ek} c_j^{*en} g_{kn}^* \tag{3}$$
$$g_{ij}^\circ = c_i^{\wedge k} c_j^{\wedge n} g_{kn}^\wedge, \quad g_{ij}^* = c_i^{\wedge ek} c_j^{\wedge en} g_{kn}^\wedge, \quad g_{ij}^\circ = c_i^{*pk} c_j^{*pn} g_{kn}^*$$
$$c_i^{\circ k} c_k^{\wedge j} = \delta_i^j, \quad c_i^{\circ pk} c_k^{*pj} = \delta_i^j, \quad c_i^{*ek} c_k^{\wedge ej} = \delta_i^j$$

where $u_i^p$, $u_i^e$ and $u_i$ are the components of the plastic, elastic and total displacement vectors. Using (1) and (2), we can write the deformation tensor components in one of the following forms:

$$2\varepsilon_{ij}^p = c_i^{\circ pk} c_j^{\circ pn} g_{kn}^\circ - g_{ij}^\circ, \quad 2\varepsilon_{ij}^e = c_i^{*ek} c_j^{*en} g_{kn}^* - g_{ij}^* \tag{4}$$
$$2\varepsilon_{ij} = c_i^{\circ k} c_j^{\circ n} g_{kn}^\circ - g_{ij}^\circ$$

The components of finite deformation are expressed, in accordance with (3), in terms of the displacements in a nonlinear manner. This throws doubts on the determination of the deformation components, since the paper [2] asserts that the additivity of the deformations does not imply the additivity of the displacements. The discrepancy is explained as follows: the last relation of (1) connects the tensor components which have different basis, although they lie in the same Lagrangian coordinate system. Indeed, the additivity of the components of the displacement vectors with different bases is not apparent. If however we reduce the displacements to a single basis, then the additivity property will hold. Indeed, the last relation of (1) can be transformed, using (4) and (3), into the equivalent expression

$$c_i^{\circ m} c_j^{\circ s} g_{ms}^{\circ} = c_i^{*ek} c_j^{*en} c_k^{\circ pm} c_n^{\circ ps} g_{ms}^{\circ} \tag{5}$$

from which we obtain

$$c_i^{\circ m} = c_i^{*ek} c_k^{\circ pm} \tag{6}$$

Transforming the tensor $\nabla_i{}^* u^{ek}$ to the basis $\vartheta_i^\circ$ yields

$$\nabla_i{}^* u^{ek} = \nabla_i u^{\circ es} c_s^{*pk}, \quad c_i^{*ek} = \delta_i^{\ k} + \nabla_i u^{*ek} \tag{7}$$

Using (7) and (3) we obtain from (6)

$$c_i^{\circ m} = (\delta_i^{\ k} + \nabla_i u^{\circ es} c_k^{*pk}) c_k^{\circ pm} = c_i^{\circ pm} + \nabla_i u^{\circ em}$$

and this yields, in accordance with the notation adopted,

$$\nabla_i{}^\circ u^m = \nabla_i{}^\circ u^{pm} + \nabla_i{}^\circ u^{em}$$

which proves that the displacements are additive.

It must be noted that the assertion given in [2] stating that the relation $F = F^e F^p$ (where $F$ is the matrix of the displacement gradients) is less general than (1), is not fully justified, although this relation is analogous to (6).

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